

# Low-momentum interactions for nuclei

Achim Schwenk

Nuclear Theory Center, Indiana University, Bloomington, IN 47408

**Abstract.** We show how the renormalization group is used to construct a low-momentum nucleon-nucleon interaction  $V_{\text{low } k}$ , which unifies all potential models used in nuclear structure calculations.  $V_{\text{low } k}$  can be directly applied to the nuclear shell model or to nucleonic matter without a  $G$  matrix resummation. It is argued that  $V_{\text{low } k}$  parameterizes a high-order chiral effective field theory two-nucleon force. We use cutoff dependence as a tool to assess the error in the truncation of nuclear forces to two-nucleon interactions and introduce a low-momentum three-nucleon force, which regulates  $A = 3, 4$  binding energies. The adjusted three-nucleon interaction is perturbative for small cutoffs. In contrast to other precision interactions, the error due to missing many-body forces can be estimated, when  $V_{\text{low } k}$  and the corresponding three-nucleon force are used in nuclear structure calculations and the cutoff is varied.

## 1. Introduction

There has been much progress over the last five years on improving many-body methods applicable to nuclei and nucleonic matter. These improvements are most successful in different regions of the nuclear chart: the Bloch-Horowitz approach for few-body systems, the No-Core Shell Model for light nuclei, the Coupled Cluster Method for the intermediate mass region, Density Functional Theory and effective actions for heavy nuclei, and the Renormalization Group approach for nucleonic matter. Although these microscopic many-body approaches are in principle different methods to diagonalize an  $A$ -nucleon Hamiltonian, an error is always introduced since it is not possible to include up to  $A$ -body forces. Thus, it is important to understand the error of the truncation, e.g., to two-nucleon (NN), or two- and three-nucleon (3N) interactions, and to explore different choices in the nuclear force starting point.

When systems are probed at low energies, it is convenient to use low-momentum degrees-of-freedom and replace the unresolved short-distance details by something simpler, without distorting low-energy observables. As a result, there are an infinite number of low-energy potentials corresponding to different resolutions, and one can use this freedom constructively to pick a convenient one. In nuclear physics, many issues depend on the resolution, e.g., the strength of 3N relative to NN forces, the spin-orbit splitting obtained from NN only, or the size of exchange correlations. The change of the resolution scale corresponds to changing the cutoff in nuclear forces, and thus this freedom is lost if one uses the cutoff as a fit parameter, or cannot vary it substantially.

In this Talk, we review results for a “universal” low-momentum NN interaction, called  $V_{\text{low } k}$ . This unifies all potential models used in nuclear structure calculations. We present Faddeev results for few-body systems and show that the cutoff variation of  $A = 3, 4$  binding energies is of the same size as results for different precision NN interactions, but low-momentum cutoffs are closer to experiment. This demonstrates that cutoff independence, and thus model independence, in nuclear physics requires consistent 3N forces. After augmenting  $V_{\text{low } k}$  by chiral 3N forces, we find that 3N contributions are perturbative for small cutoffs. The set of low-momentum two- and three-nucleon interactions can be used in calculations of nuclear structure and reactions and we discuss promising directions. Finally, we show that  $V_{\text{low } k}$  and  $G$  matrix elements are quantitatively similar, but  $V_{\text{low } k}$  as a potential has a solid theoretical foundation with corresponding 3N forces, whereas a  $G$  matrix introduces uncontrolled approximations.

## 2. Low-momentum nucleon-nucleon interaction

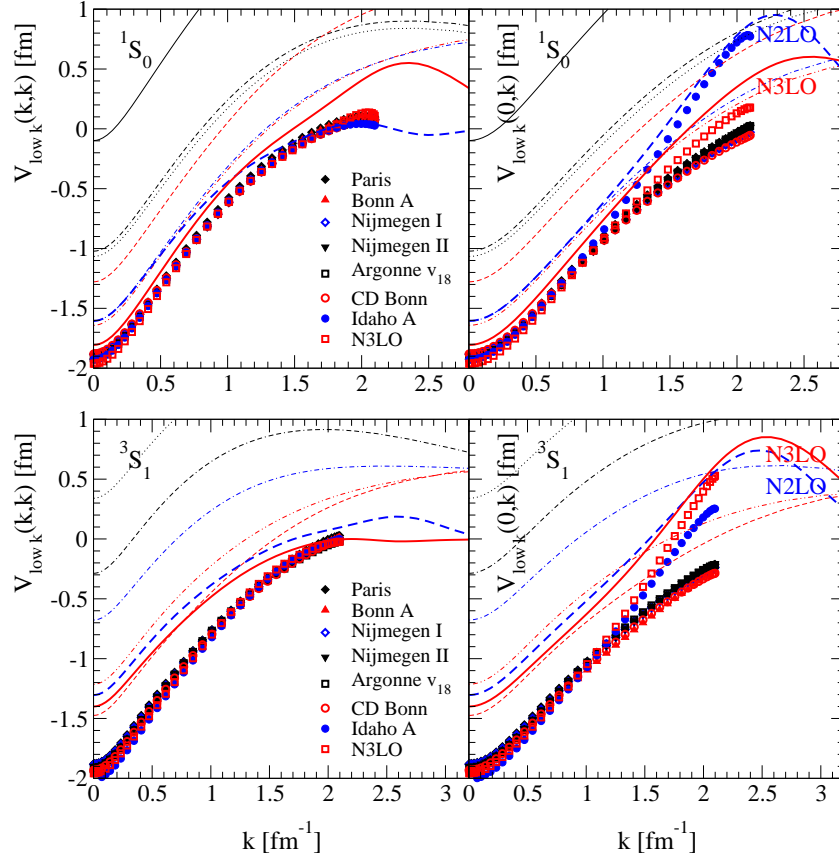
Conventional precision NN interactions are well-constrained by two-nucleon scattering data only for laboratory energies  $E_{\text{lab}} \lesssim 350$  MeV. As a consequence, details of nuclear forces are not constrained for relative momenta  $k > 2.0 \text{ fm}^{-1}$  or distances  $r < 0.5$  fm. However, all these potentials have strong high-momentum components as illustrated by the different lines in Fig. 1. This leads to model dependences and technical difficulties in many-body applications. Starting from a given potential model  $V_{\text{NN}}$ , we have integrated out the high-momentum modes above a cutoff  $\Lambda$  in the sense of the renormalization group (RG) [1, 2]. The resulting low-momentum interaction  $V_{\text{low } k}$  only has momentum components below the cutoff and evolves with  $\Lambda$  so that the low-momentum scattering amplitude  $T(k', k; k^2)$  (in particular phase shifts and deuteron binding energy) are invariant. Thus, in every scattering channel we have

$$T(k', k; k^2) = V_{\text{NN}}(k', k) + \frac{2}{\pi} \mathcal{P} \int_0^\infty \frac{V_{\text{NN}}(k', p) T(p, k; k^2)}{k^2 - p^2} p^2 dp, \quad (1)$$

$$T(k', k; k^2) = V_{\text{low } k}^\Lambda(k', k) + \frac{2}{\pi} \mathcal{P} \int_0^\Lambda \frac{V_{\text{low } k}^\Lambda(k', p) T(p, k; k^2)}{k^2 - p^2} p^2 dp. \quad (2)$$

In order to reproduce the low-momentum  $T$  matrix for a given cutoff,  $V_{\text{low } k}$  is renormalized for scattering to intermediate states with  $p > \Lambda$ . This is achieved by resumming high-momentum ladders in an energy-dependent effective interaction, which is the solution to the two-body Bloch-Horowitz equation in momentum space with projector  $Q = \theta(p - \Lambda)$ . The energy dependence can then be recast as momentum dependence by using the equations of motion. Both steps are equivalent to the basis transformation of Lee-Suzuki.† We note that the largest effect of the renormalization is due to the first step of integrating out the high-momentum modes (for laboratory energies  $E_{\text{lab}} \lesssim 150$  MeV the zero-energy Bloch-Horowitz potential describes the phase

† Note that the RG approach differs from Lee-Suzuki, as we set the  $Q$  space block of the effective Hamiltonian (which includes all model dependences) to zero.



**Figure 1.** Diagonal (left) and off-diagonal (right) momentum-space matrix element for  $V_{\text{low } k}$  (symbols) versus relative momentum derived from different high-precision potential models for  $\Lambda = 2.1 \text{ fm}^{-1}$ . The various bare interactions are given as lines, with thick solid or thick dashed lines for the N2LO (Idaho A) or N3LO interactions respectively. Results are shown for the  $^1S_0$  (upper) and  $^3S_1$  partial wave (lower figures).

shifts accurately). Both steps are equivalent to integrating an RG equation [1]

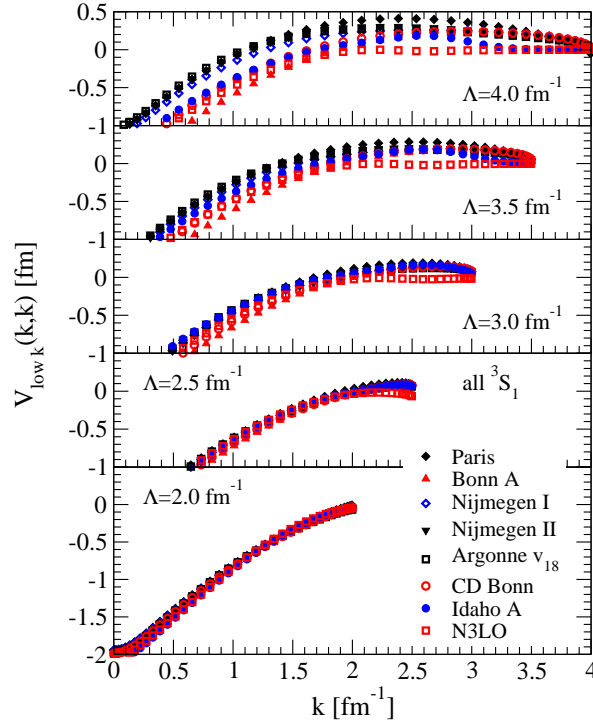
$$\frac{d}{d\Lambda} V_{\text{low } k}^\Lambda(k', k) = \frac{2}{\pi} \frac{V_{\text{low } k}^\Lambda(k', \Lambda) T^\Lambda(\Lambda, k; \Lambda^2)}{1 - (k/\Lambda)^2}. \quad (3)$$

For every cutoff  $V_{\text{low } k}$  defines a new NN potential and a new low-momentum Hamiltonian

$$H_{\text{low } k}^\Lambda = T + \bar{V}_{\text{low } k}^\Lambda, \quad (4)$$

where the cutoff acts only on the interaction and  $\bar{V}_{\text{low } k}$  denotes a (Okubo-) Hermitized  $V_{\text{low } k}$  (from now on all  $V_{\text{low } k}$  results are for the Hermitian  $\bar{V}_{\text{low } k}$  and we drop the overline). In many-body applications,  $H_{\text{low } k}^\Lambda$  will lead to different results from  $T + V_{\text{NN}}$  (since unresolved interactions between any high-momentum nucleons are excluded).

Our main results are shown in Fig. 1. By performing an RG decimation to  $\Lambda \lesssim 2.1 \text{ fm}^{-1}$ , we find that all NN potentials that fit the scattering data and include the same long-distance pion physics lead to a “universal” low-momentum interaction  $V_{\text{low } k}$  [1, 2]. This holds for all channels and low-momentum off-diagonal matrix elements. Note also that in the  $^1S_0$  channel, where there is a significant change of  $V_{\text{NN}}$  from N2LO to N3LO, the  $V_{\text{low } k}$  moves towards the “universal” curve from N2LO to N3LO. Finally,



**Figure 2.** Evolution of the diagonal  $V_{\text{low } k}$  matrix elements obtained from different potentials for cutoffs  $\Lambda = 2.0 \dots 4.0 \text{ fm}^{-1}$  versus relative momentum in the  $^3S_1$  channel.

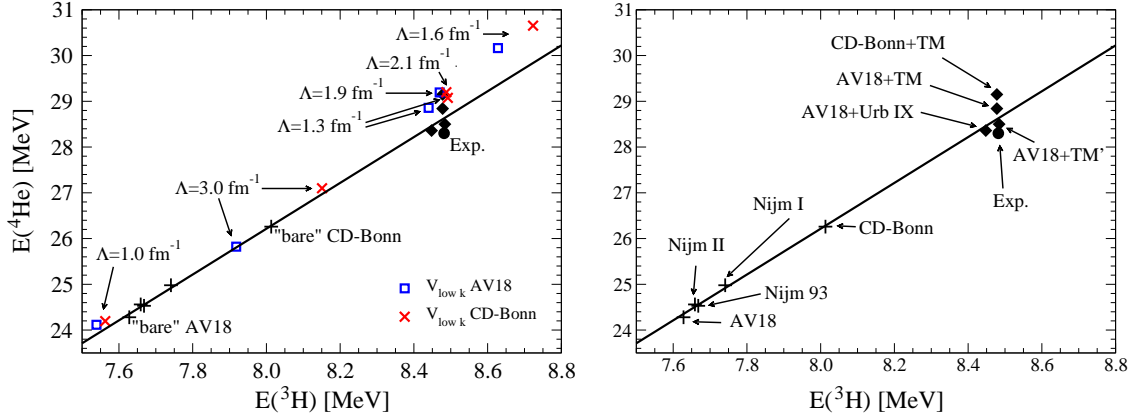
we illustrate the collapse in Fig. 2, where we show the evolution of diagonal  $V_{\text{low } k}$  matrix elements from  $\Lambda = 2.0 \dots 4.0 \text{ fm}^{-1}$ . Further results and details can be found in [2].

We emphasize that the renormalization of high-momentum modes is theoretically and in practice easier in free space, before going to a many-body system.<sup>§</sup>  $V_{\text{low } k}$  does not require a  $G$  matrix resummation, which was introduced because of (model-dependent) high-momentum modes in nuclear forces. Finally,  $V_{\text{low } k}$  is energy-independent and the cutoff is not a parameter (no “magic” value). As we demonstrate in the next Section, the cutoff can be used to assess the error of a Hamiltonian truncated to two-body forces.

### 3. Cutoff dependence as a tool to assess missing many-body forces

All NN interactions have a cutoff (“P-space of QCD”) and therefore have corresponding three- and higher-body forces. Consequently, if one omits the many-body forces, 3N, 4N,... observables will be cutoff-dependent. Using  $V_{\text{low } k}$  all low-energy NN observables are cutoff-independent, and therefore, we can assess the effects of the omitted 3N, 4N,... forces by varying the cutoff in many-body calculations. In Fig. 3, we present results

<sup>§</sup> We also note that the RG evolution is very useful for chiral effective field theory (EFT) interactions. This is because for lower cutoffs, the phase space for intermediate states is smaller in nuclear structure applications. We can start from a chiral EFT interaction with cutoff range, e.g.,  $\Lambda_\chi \sim 500 - 700 \text{ MeV}$  to include the maximum known long-distance physics, and then run the cutoff down lower. Observables are preserved under the RG and higher-order operators are induced automatically, which is more accurate and faster than fitting a chiral EFT truncation at the lower cutoff.



**Figure 3.** Correlation of the triton and alpha particle binding energies. We contrast the  $V_{\text{low } k}$  results (left figure) to results for several modern potential models (right figure, taken from [3], where plusses denote NN only and diamonds NN with adjusted 3N forces). Results are given for the  $V_{\text{low } k}$  derived from the Argonne  $v_{18}$  or the CD Bonn potential. The Tjon line is shown as a linear fit to the NN-only results. The thick solid lines in the left figure are interpolations from the Argonne  $v_{18}$ /CD Bonn results to the respective  $V_{\text{low } k}$  for the largest cutoff studied.

for the  ${}^3\text{H}$  and  ${}^4\text{He}$  binding energies for a wide range of cutoffs [4]. We find that the cutoff variations for all  $\Lambda \geq 1.0 \text{ fm}^{-1}$  are approximately 1 MeV and 5 MeV for the  $A = 3$  and  $A = 4$  binding energies respectively. The variation should be compared to the potential energy, which is  $\langle V_{\text{low } k}^\Lambda \rangle > 30 \text{ MeV}$  and  $> 60 \text{ MeV}$  in the two systems. The variation gives an estimate of the contribution from many-body forces and shows that, while retaining a  $\chi^2/\text{datum} \approx 1$ , the truncation to a NN potential alone will have associated errors of 1 MeV and 5 MeV for  ${}^3\text{H}$  and  ${}^4\text{He}$ . We also find that the cutoff dependence predicts the linear correlation between binding energies known as the Tjon line. Results for reasonable low-momentum cutoffs  $\Lambda \sim 2.0 \text{ fm}^{-1}$  are closer to experiment and we observe a slight breaking off the Tjon line. Our results demonstrate that 3N forces are inevitable for renormalization and are similar to results obtained in the pionless EFT (see proceedings by H.-W. Hammer). Two reference results for the  $V_{\text{low } k}$  obtained from the Argonne  $v_{18}$  potential and  $\Lambda = 1.9 \text{ fm}^{-1}$  are  $E({}^3\text{H}) = 8.47 \text{ MeV}$  and  $E({}^4\text{He}) = 29.19 \text{ MeV}$  (for the  $V_{\text{low } k}$  derived from the CD Bonn potential and  $\Lambda = 2.1 \text{ fm}^{-1}$ , we have  $E({}^3\text{H}) = 8.49 \text{ MeV}$  and  $E({}^4\text{He}) = 29.20 \text{ MeV}$ ).

In the right part of Fig. 3, we also give the potential model dependence in  $A = 3, 4$  systems. This model dependence is due to the probing of the unconstrained high-momentum modes. The high-momentum modes induce three-body correlations, which are of short-range. Therefore, at low energies these effects are inseparable from the effects due to omitted 3N interactions. Finally, we note that Fujii *et al.* have reported similar results for  $V_{\text{low } k}$  [5], but conclude that large cutoffs should be used. However, this conclusion is misguided, because it relies on reproducing the few-body binding energies of a specific  $V_{\text{NN}}$ -only model with  $V_{\text{low } k}$  (both without the corresponding 3N forces). Since for every cutoff,  $V_{\text{low } k}^\Lambda$  is a new potential, the few-body binding energies will be different, as expected from the right part of Fig. 3.

$\Lambda$	${}^3\text{H}$					${}^4\text{He}$					max	${}^4\text{He}$
	$T$	$V_{\text{low } k}$	$c$ -terms	$D$ -term	$E$ -term	$T$	$V_{\text{low } k}$	$c$ -terms	$D$ -term	$E$ -term	$ V_{3\text{N}}/V_{\text{low } k} $	$k_{\text{rms}}$
1.0	21.06	-28.62	0.02	0.11	-1.06	38.11	-62.18	0.10	0.54	-4.87	0.08	0.55
1.3	25.71	-34.14	0.01	1.39	-1.46	50.14	-78.86	0.19	8.08	-7.83	0.10	0.63
1.6	28.45	-37.04	-0.11	0.55	-0.32	57.01	-86.82	-0.14	3.61	-1.94	0.04	0.67
1.9	30.25	-38.66	-0.48	-0.50	0.90	60.84	-89.50	-1.83	-3.48	5.68	0.06	0.70
2.5( <i>a</i> )	33.30	-40.94	-2.22	-0.11	1.49	67.56	-90.97	-11.06	-0.41	6.62	0.12	0.74
2.5( <i>b</i> )	33.51	-41.29	-2.26	-1.42	2.97	68.03	-92.86	-11.22	-8.67	16.45	0.18	0.74
3.0(*)	36.98	-43.91	-4.49	-0.73	3.67	78.77	-99.03	-22.82	-2.63	16.95	0.23	0.80

**Table 1.** Expectation values of the kinetic energy ( $T$ ),  $V_{\text{low } k}$  and the different 3N contributions (long-range  $2\pi$ -exchange ( $c$ -terms),  $1\pi$ -exchange part ( $D$ -term) and contact interaction ( $E$ -term)) for  ${}^3\text{H}$  and  ${}^4\text{He}$ . All energies are in MeV and momenta are in  $\text{fm}^{-1}$ . (*a*) and (*b*) denote two possible solutions for  $\Lambda = 2.5 \text{ fm}^{-1}$  and (\*) indicates that the  ${}^4\text{He}$  fit is approximate, for details see [4]. In addition, we give the ratio of maximum 3N to  $V_{\text{low } k}$  contribution and an average relative momentum  $k_{\text{rms}}$ .

#### 4. Perturbative low-momentum three-nucleon interaction

It would be extremely nice to use the RG and calculate a low-momentum NN and 3N interaction from different potential models. This however has two problems. First, it requires accurate calculations of high-energy 3N scattering wave functions. Second, and more severe, with the exception of chiral EFT interactions there is no consistency between the NN potential models and their fitted 3N forces. As a consequence, we have decided to adjust the leading-order chiral 3N forces to  $V_{\text{low } k}$  for various cutoffs. The motivation for this is that at low energies, all phenomenological 3N forces due to meson exchanges and high-momentum N,  $\Delta$ ,... intermediate states collapse to this operator form. Therefore, it is reasonable to use the operator form constrained by chiral EFT and adjust the coupling constants to  $V_{\text{low } k}$ . Moreover, cutoffs in  $V_{\text{low } k}$  and chiral potentials are very similar. Both are low-momentum interactions, where only pion exchanges are explicitly resolved. When we start from chiral interactions and run the cutoff down lower, we find that they also collapse to the “universal”  $V_{\text{low } k}$ , which indicates that  $V_{\text{low } k}$  parameterizes a higher-order EFT interaction with sharp-cutoff regularization.

In chiral EFT, the leading-order 3N interaction enters at N<sup>2</sup>LO and consists of a long-range  $2\pi$ -exchange, an intermediate-range  $1\pi$ -exchange part and a short-range contact interaction [6, 7]. There are five coupling constants: three low-energy  $c$  constants in the  $2\pi$  part, as well as  $D$ - and  $E$ -term couplings in the  $1\pi$  and contact term respectively. A possible determination of the  $c$  constants is through a NN partial wave analysis, which includes the long-distance  $1\pi$  and  $2\pi$  physics in the interaction. This has been carried out by the Nijmegen group, and we take their values for the  $c$  constants [8], which is most in keeping with our results that  $V_{\text{low } k}$  is strongly constrained by the scattering data. Since the  $c$  constants parameterize low-energy  $\pi N$  physics, their values are independent of the cutoff used to regularize nuclear forces.

We then adjust the two  $D$ - and  $E$ -term couplings to the  ${}^3\text{H}$  and  ${}^4\text{He}$  binding energies for different cutoffs (We note that it may be better to adjust the 3N force to  ${}^3\text{H}$  and a heavier system, say  ${}^{16}\text{O}$ , when the latter can be calculated more accurately in the

future). For cutoffs  $\Lambda \lesssim 2.0 \text{ fm}^{-1}$  we find linear dependences in the fitting, which are consistent with a perturbative  $D$ - and  $E$ -term contribution  $E(^3\text{H}) = E(V_{\text{low } k} + c\text{-terms}) + c_D \langle D\text{-term} \rangle + c_E \langle E\text{-term} \rangle$  ( $c_D$  and  $c_E$  are the coupling constants and  $\langle \dots \rangle$  denote the matrix elements of the operators). This has been checked explicitly and also for the  $c$ -terms. For  $V_{\text{low } k}$  and all studied cutoffs  $\Lambda \lesssim 2.0 \text{ fm}^{-1}$ , we find that the adjusted 3N forces are perturbative [4], by which we mean  $\langle \Psi^{(3)} | V_{3\text{N}} | \Psi^{(3)} \rangle \approx \langle \Psi^{(2)} | V_{3\text{N}} | \Psi^{(2)} \rangle$ , where  $|\Psi^{(n)}\rangle$  are exact solutions including up to  $n$ -body forces. We use the operator form of the chiral 3N force given in Eq. (2) and Eq. (10) in [7], multiplied by an exponential regulator  $\exp[-((p^2 + 3q^2/4)/\Lambda^2)^4]$  with the same cutoff value as in  $V_{\text{low } k}$  ( $p$  and  $q$  are Jacobi momenta). The high power in the exponent yields a behavior similar to a sharp cutoff. The two parameters of the 3N force fit to  $V_{\text{low } k}$  are tabulated for a wide range of cutoffs in [4]. We note that it is non-trivial that a fit solution of the leading-order chiral 3N form with realistic  $c$  constants exists.

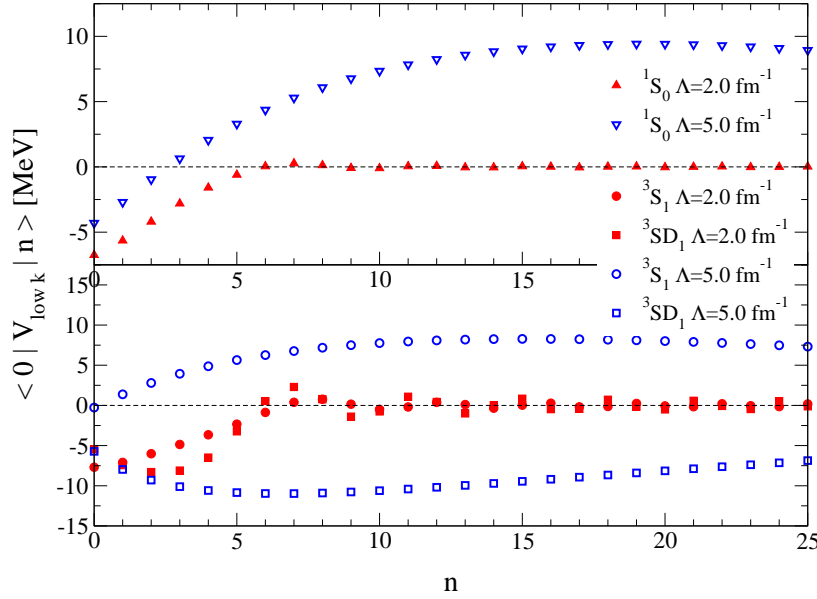
Next, we present the different contributions to the triton and alpha particle binding energies in Table 1. Assuming that the kinetic energy is due to independent particle pairs, we can use this to obtain an average relative momentum  $k_{\text{rms}} = \sqrt{\langle k^2 \rangle} \approx mT/(A-1)$ , when  $V_{\text{low } k}$  is used in these systems. Over the range of cutoffs in Table 1, we find  $k_{\text{rms}} \approx 0.55 \dots 0.80 \text{ fm}^{-1}$  for  $^4\text{He}$  ( $0.50 \dots 0.67 \text{ fm}^{-1}$  for  $^3\text{H}$ ). Although not observable, it is reassuring that  $k_{\text{rms}} \ll \Lambda$  and also intriguing that for all low-momentum cutoffs  $k_{\text{rms}} \sim m_\pi$ , as expected in chiral EFT. We also find that the non-linearities in the fitting for larger cutoffs  $\Lambda \gtrsim 2.5 \text{ fm}^{-1}$  lead to a ratio of the maximum 3N to  $V_{\text{low } k}$  contribution  $\approx 0.2$ . In the chiral counting, 3N contributions are on the order of  $(Q/\Lambda)^3$  relative to the NN force, where  $Q$  is a typical momentum in the system. With  $Q \sim k_{\text{rms}} \sim m_\pi$ , we find  $(Q/\Lambda)^3 \approx 0.05$  for  $\Lambda \sim 2.0 \text{ fm}^{-1}$  and thus the 0.2 ratio at larger cutoffs is beyond this expectation. We take this as an indication that the leading-order chiral 3N force is insufficient for larger cutoffs, where more physics is resolved, or that one enters the non-linear regime of a limit cycle (see proceedings by A. Nogga). We also observe that the  $c$ -terms and the  $E$ -term increase with increasing cutoff and cancel. This is expected since the  $E$ -term renormalizes all divergences in the 3N system.

It is important to note that the 3N contributions, while perturbative for all cutoffs  $\Lambda \lesssim 2.0 \text{ fm}^{-1}$ , increase by a factor  $\sim 5$  from  $A = 3$  to  $A = 4$ . This density dependence leads to saturation in nuclear matter [9]. Finally, for further details on  $V_{\text{low } k}$  in few-nucleon systems and the low-momentum 3N force see [4] and proceedings by A. Nogga.

## 5. Harmonic-oscillator matrix elements and $G$ matrix comparison

The advantage of using low-momentum interactions in many-body applications is that  $V_{\text{low } k}$  is a soft interaction, without a strong core at short distances. Therefore,  $V_{\text{low } k}$  does not couple strongly to high-lying states and can be used in small model spaces. This makes it for the first time possible to start directly from a precision NN interaction for applications in nuclear structure or reactions.

The benefit of a lower cutoff in shell model applications is shown in Fig 4, where

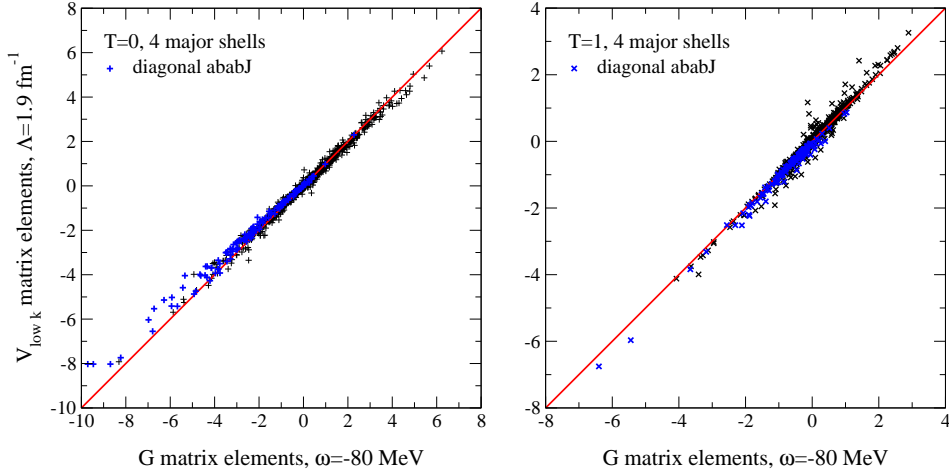


**Figure 4.** Relative harmonic-oscillator matrix elements  $\langle n' = 0 | V_{\text{low } k} | n l S J \rangle$  versus radial quantum number  $n$  for a low-momentum cutoff  $\Lambda = 2.0 \text{ fm}^{-1}$  and a 1.0 GeV cutoff  $\Lambda = 5.0 \text{ fm}^{-1}$ . Results are shown for the S-wave matrix elements. In both cases,  $V_{\text{low } k}$  is obtained from the Argonne  $v_{18}$  potential and  $\hbar\omega = 14 \text{ MeV}$ .

we compare S-wave relative harmonic-oscillator matrix elements for a low-momentum cutoff  $\Lambda = 2.0 \text{ fm}^{-1}$  and a 1.0 GeV cutoff  $\Lambda = 5.0 \text{ fm}^{-1}$ . We find that for low-momentum interactions the matrix elements decrease quite rapidly and become small for  $|n - n'| \sim 10$ . This is not the case for interactions with high-momentum components, which require basis states up to  $\sim 50$  shells for convergence. Fig. 4 clearly shows that strong high-momentum modes are only poorly represented in a shell model basis.

In conventional approaches to shell-model effective interactions, the cores are tamed by performing a ladder resummation of a  $V_{\text{NN}}$  model to obtain a  $G$  matrix. However, the  $G$  matrix resummation introduces an uncontrolled starting-energy dependence and requires further approximations in practice. Moreover, there is no theory for the starting energy, since the Bloch-Horowitz energy self-consistency is lost when one restricts the effective interaction to two-body in an  $A$ -body system. In Fig. 5, we compare  $V_{\text{low } k}$  to  $G$  matrix elements in four major shells. We find that in both  $T = 0, 1$  channels the matrix elements are very similar. The biggest differences are on the diagonal  $T = 0$  matrix elements. This comes as no surprise since the diagonal matrix elements are related to the monopole interaction, responsible for most of the nuclear binding. Here, we expect the largest effect of the low-momentum 3N force. A detailed and more quantitative comparison will be presented in [11], where we also study the phenomenology of low-momentum interactions and point out where calculations including 3N forces are most needed. We note that similar correlations between  $V_{\text{low } k}$  and a  $G$  matrix hold for different cutoffs and different  $\hbar\omega$  (up to a simple scaling) [11]. Here, we only want to show that  $V_{\text{low } k}$  can be used directly in nuclear structure applications, and emphasize that  $V_{\text{low } k}$  has a well-defined theoretical basis with perturbative, low-momentum 3N forces.





**Figure 5.** Correlation plots between  $V_{\text{low } k}$  and  $G$  matrix elements in 4 major shells. The matrix elements  $\langle ab | \dots | cdJT \rangle$  are in MeV for  $\hbar\omega = 14$  MeV. We also distinguish the diagonal elements related to the monopole interaction and thus nuclear binding.  $V_{\text{low } k}$  is derived from the Argonne  $v_{18}$  potential and the  $G$  matrix is for Idaho A computed with a rectangular Pauli operator for 4 major shells and starting energy  $-80$  MeV [10] (for other correlation plots see proceedings by B.A. Brown).

## 6. Summary and outlook

In this Talk, we have reviewed advances in constructing low-momentum interactions, which can be used directly in many-body applications. Our results show that, for low-momentum cutoffs, nuclear forces are well constrained and that difficult-to-handle cores are not needed to reproduce the NN scattering data. We have shown how the RG can be used to construct a model-independent low-momentum interaction, which unifies all precision potential models used in nuclear structure calculations. We believe that  $V_{\text{low } k}$  is a very useful for nuclear many-body problems for the following reasons:

- (i) It is possible to vary the cutoff in  $V_{\text{low } k}$  (or  $V_{\text{low } k}$  and adjusted 3N interactions) over a wide range. This enables one to estimate an error due to omitted many-body forces (or omitted higher-order 3N, 4N,... interactions). In this way, it is possible to vary the cutoff for interactions with singular pion exchanges and obtain an error control in many-body calculations as in the pionless EFT.
- (ii) We have shown that 3N forces are required by renormalization and that adjusted low-momentum 3N interactions are perturbative for cutoffs  $\Lambda \lesssim 2.0 \text{ fm}^{-1}$ . This should considerably simplify including 3N forces in nuclear structure applications, e.g., shell model interactions, coupled cluster theory or nucleonic matter.
- (iii) Low-momentum interactions bind nuclei in Hartree-Fock [12], in contrast to all other microscopic NN interactions. Consequently, exchange correlations are smaller starting from low-momentum forces, and a quantitative derivation of a nuclear density functional seems possible [13]. Moreover, preliminary results indicate that nuclear matter with  $V_{\text{low } k}$  and 3N forces is perturbative [9], for neutron matter the

Hartree-Fock equation of state is very reasonable [14].

- (iv)  $V_{\text{low } k}$  as a potential provides a well-defined starting point for microscopic calculations of effective interactions for heavier nuclei in small model spaces.

Applications of  $V_{\text{low } k}$  that were not discussed in this Talk range from valence particle nuclei [15] to quasiparticle interactions and pairing in neutron matter [16, 17].

We close with a recommendation of how to start using  $V_{\text{low } k}$  and some priorities for future research. If one wants to use  $V_{\text{low } k}$  as a new potential without varying the cutoff, we suggest to use cutoffs near  $\Lambda = 2.0 \text{ fm}^{-1}$  (for the  $V_{\text{low } k}$  derived from the Argonne  $v_{18}$  (CD Bonn) potential the triton binding energy is accidentally reproduced for  $\Lambda = 1.9 \text{ fm}^{-1}$  ( $\Lambda = 2.1 \text{ fm}^{-1}$ )). We would take these as first cutoff values but stress that the 3N force never vanishes. For future investigation, it would be extremely promising to study the cutoff variation of nuclear spectra and convergence properties using  $V_{\text{low } k}$  with 3N forces in the No-Core Shell Model or Coupled Cluster Theory. Varying the cutoff will be a powerful tool to provide theoretical error estimates for extrapolations towards the drip lines, where one cannot compare to experiment. A chart for the spectra of light nuclei with theoretical error bars would be wonderful. Finally, more insight on the effects of 3N forces will come from shell model calculations using  $V_{\text{low } k}$  with perturbative 3N forces, especially where a two-body  $G$  matrix fails.

It is a pleasure to thank my collaborators Scott Bogner, Gerry Brown, Bengt Friman, Dick Furnstahl, Chuck Horowitz, Tom Kuo, Andreas Nogga, Janos Polonyi and Andres Zuker for many discussions. This work is supported by the DOE under grant No. DEFG 0287ER40365 and the NSF under grant No. nsf-phy 0244822.

## References

- [1] S.K. Bogner *et al.*, Phys. Lett. **B576** (2003) 265, for details on the RG see nucl-th/0111042.
- [2] S.K. Bogner, T.T.S. Kuo and A. Schwenk, Phys. Rep. **386** (2003) 1.
- [3] A. Nogga, H. Kamada and W. Glöckle, Phys. Rev. Lett. **85** (2000) 944.
- [4] A. Nogga, S.K. Bogner and A. Schwenk, Phys. Rev. **C** (R) in press, nucl-th/0405016.
- [5] S. Fujii *et al.*, Phys. Rev. **C70** (2004) 024003.
- [6] U. van Kolck, Phys. Rev. **C49** (1999) 2932.
- [7] E. Epelbaum *et al.*, Phys. Rev. **C66** (2002) 064001.
- [8] M.C.M. Rentmeester, R.G.E. Timmermans and J.J. de Swart, Phys. Rev. **C67** (2003) 044001.
- [9] S.K. Bogner, A. Schwenk, R.J. Furnstahl and A. Nogga, in prep.
- [10] D.J. Dean and M. Hjorth-Jensen, private communication.
- [11] A. Schwenk and A. Zuker, in prep.
- [12] L. Coraggio *et al.*, Phys. Rev. **C68** (2003) 034320 and nucl-th/0407003.
- [13] A. Schwenk and J. Polonyi, nucl-th/0403011 and in prep.
- [14] A. Schwenk, G.E. Brown and B. Friman, Nucl. Phys. **A703** (2002) 745, see also nucl-th/0411070.
- [15] S.K. Bogner *et al.*, Phys. Rev. **C65** (2002) 051301(R).
- [16] A. Schwenk, B. Friman and G.E. Brown, Nucl. Phys. **A713** (2003) 191, see also nucl-th/0302081.
- [17] A. Schwenk and B. Friman, Phys. Rev. Lett. **92** (2004) 082501.